# Oscillatory Motion

and

# Simple Harmonic Motion

# Oscillatory Motion

- A repetitive back and forth motion
    - wings flapping
    - strings vibrating
    - electrical current (AC)
    - other examples?

- one complete oscillation is called a cycle

# Period and Frequency

**Recall the term period from UCM:**

**Period: The amount of time it takes to complete one revolution (in unit 3, a circle).**

**Oscillatory Motion is similar to UCM in that it shares this term:**

**Period: the amount of time it takes to complete one cycle (in this case, one back and forth movement).**

**The period of many types of oscillatory motion is very small :**

| Object | Period |
| --- | --- |
| Bumblebee wings | 0.005 s |
| Hummingbird Wings | 0.0128 s |
| Middle C on a piano | 0.004 s |
| AC Current | 0.0167 s |

**For this reason, it is often helpful to think in terms of oscillations or cycles per second. This is the called frequency.**

# Frequency

- the number of cycles per second of oscillatory motion

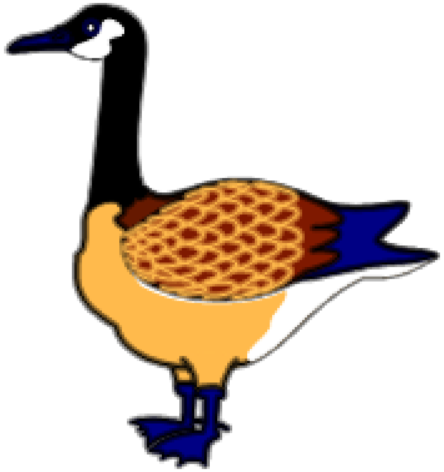
$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

where:

f = frequency in hertz (Hz) or cycles/second or $s^{-1}$

T = period (seconds)



**ex) The wings of a Canada Goose flap 200 times per minute.**

**a) What is the frequency of the flap?**

**b) What is the period of the flap?**

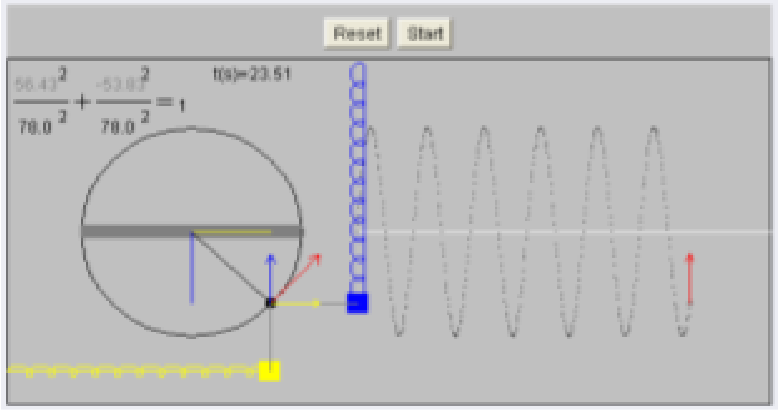
**An object which oscillates needs a force to keep it moving. Sometimes that force is supplied from outside the system (such as the muscles of a bird). Other times, that force comes from within a system.**

**The force which keeps the oscillations going is called a restoring force.**

**Any oscillatory system which has a restoring force acting against the displacement to keep an object moving is called a simple harmonic oscillator and is said to exhibit simple harmonic motion.**

**One example of a simple harmonic oscillator is a mass attached to an ideal horizontal spring sliding along a frictionless surface.**

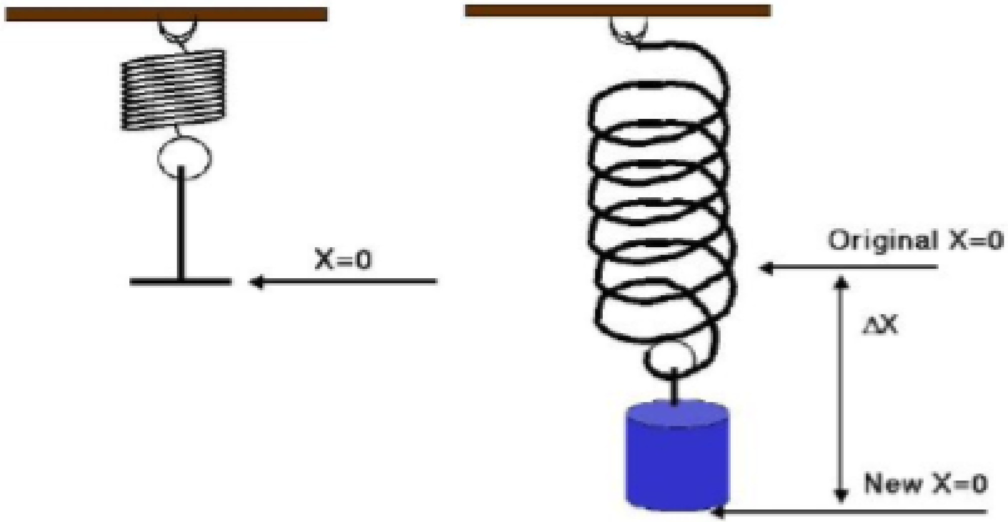

**In this situation, the spring provides the restoring force to keep the mass moving back and forth.**



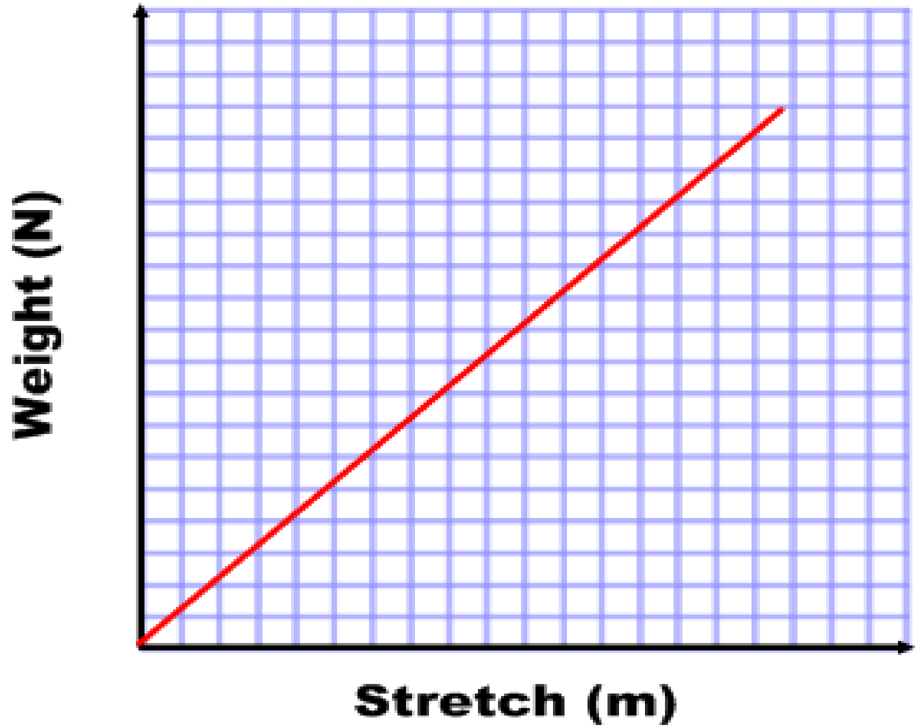

http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=148

# Hooke's Law and Elastic Energy

**In 1676, Robert Hooke devised a relationship between the amount of stretch in a spring and the weight suspended by that spring.**



When a graph of weight vs. stretch was made, a linear relationship is established.




This graph is a linear function of y-int = 0.

**Hooke's Law**

$$\vec{F} = k\vec{x}$$

where: F = force applied to the spring (N)
x = displacement from equilibrium (stretch or compress) of spring (m)
k = spring constant (slope) (N/m)

# Note:

**Hooke's Law appears with a negative sign in front of the spring constant on your formula sheet:**

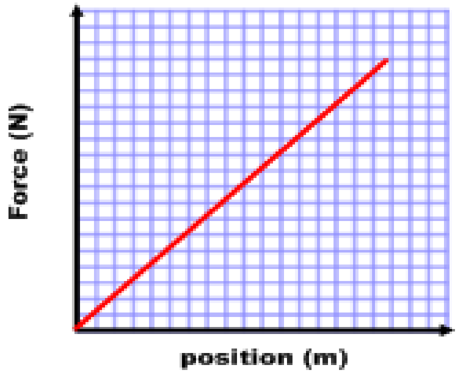
$$\vec{F}_s = -k\vec{x}$$

**because the force applied by a spring is always in the opposite direction of the displacement.**

**Hooke's discovery leads us to a way to calculate the potential energy stored in a spring or any elastic device:**

**We can not derive an equation from our usual starting statement of $W = \vec{F}\vec{d}$ because the force acting on a spring over several different masses is not constant.**

**We can, however, determine the work (and therefore energy) by finding the area under our weight vs. stretch graph.**




To calculate the area under this graph, we use:

$$A = 1/2\ bh$$

or

$$E = 1/2\vec{F}\vec{x}$$

Since our force can be described by Hooke's Law, we can sub in

$$E = 1/2\vec{F}\vec{x}$$

$$E = 1/2(\vec{k}\vec{x})\vec{x}$$

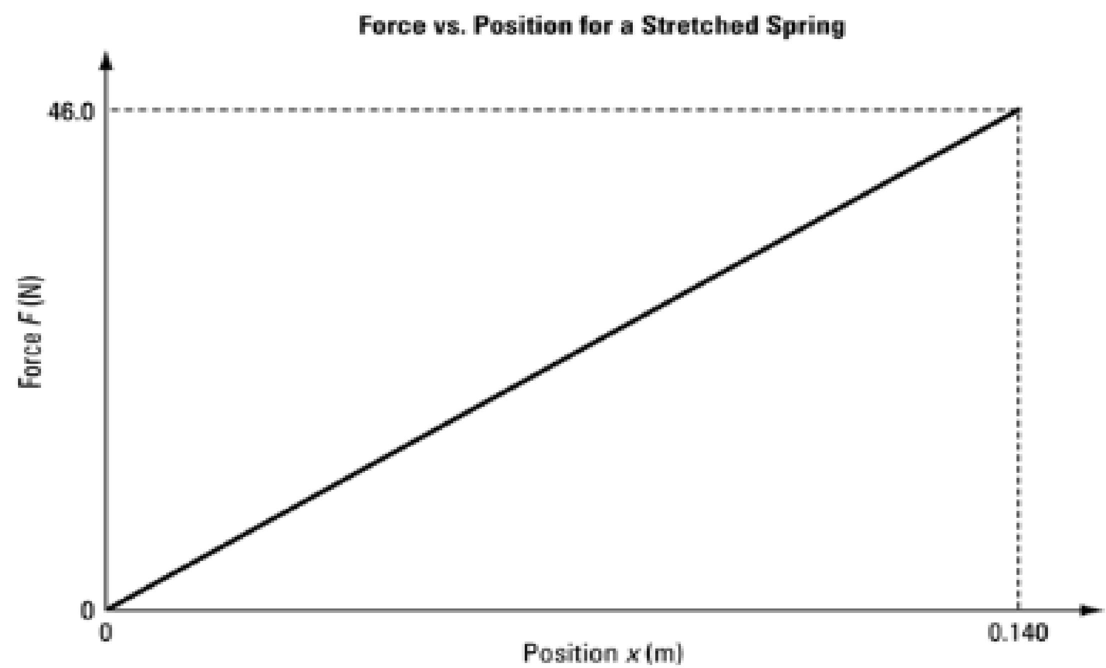
$$E_p = 1/2\vec{k}\vec{x}^2$$

elastic potential energy

Where:

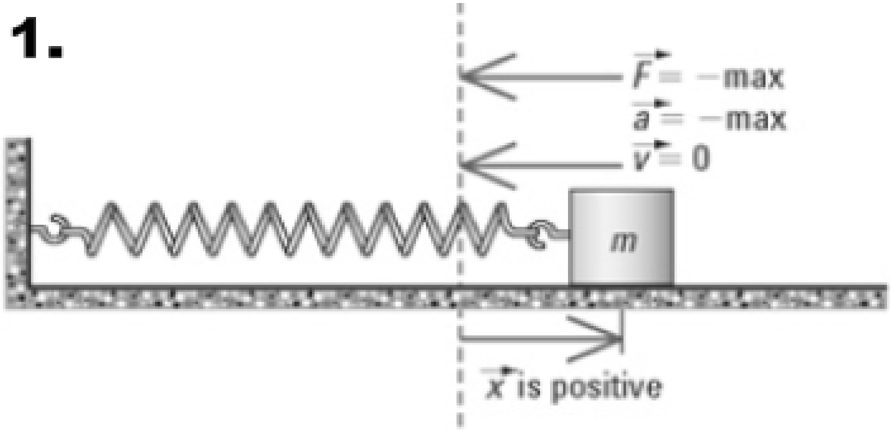
$E_p$ = Potential Energy (J)
k = Spring Constant (N/m)
x = Displacement of Spring (m)

**ex) The following graph shows how a force causes change in position as it stretches a spring.**
**Note that the force is acting parallel to the change in position.**
**(a) Calculate the energy stored in the spring when the force is 46.0 N.**
**(b) Compare the energy when the force is 46.0 N to the energy stored in the spring when its position is 7.00 cm.**



We can take a closer look at what is happening to this mass over different positions of its movement.

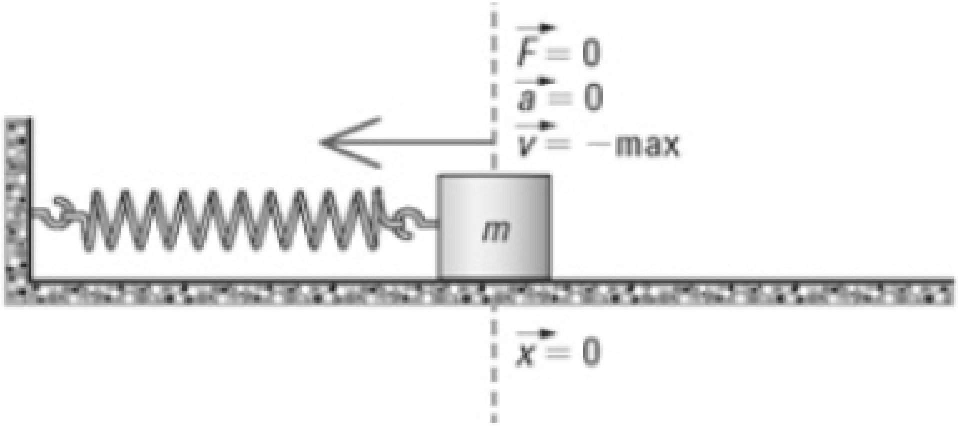
1.




The spring is stretched by a distance of $+\vec{x}$.

- mass is pulled back as far as it can go (it will return to this initial distance each cycle)
- the max. amount of stretch is called the spring's <u>amplitude</u>
- the amount of force in the spring is directly proportional to the amplitude
- when released, the restoring force of the spring will cause mass to accelerate and move towards its starting (equilibrium) position

**2.**



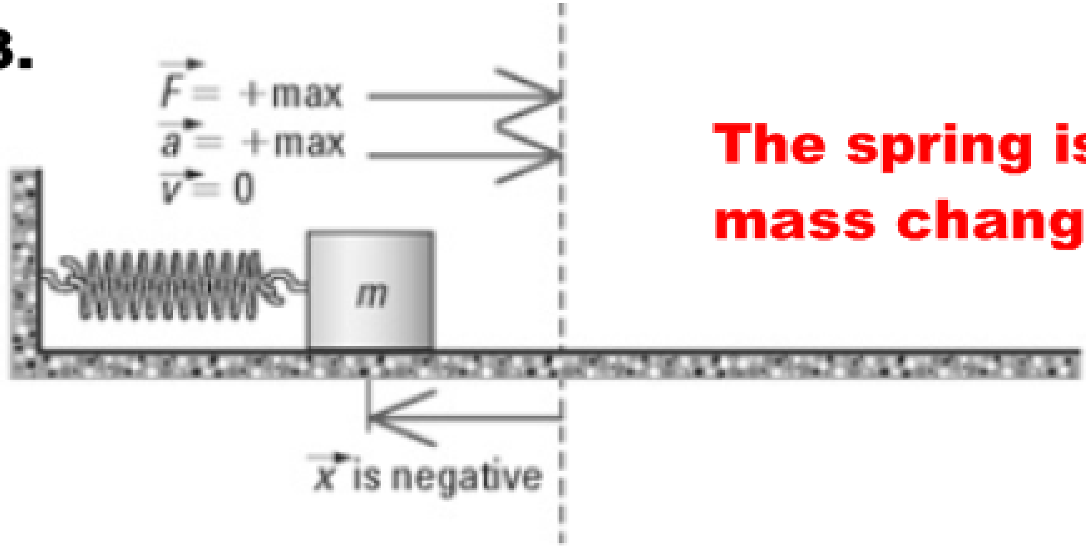

**The mass is at its maximum velocity at its equilibrium point.**

**- the spring is at the in-between moment: it is no longer stretched and has yet to be compressed**

**- no stretch in spring = no force and no acceleration**

**- the mass will continue to move left, slowing down as it compresses the spring**

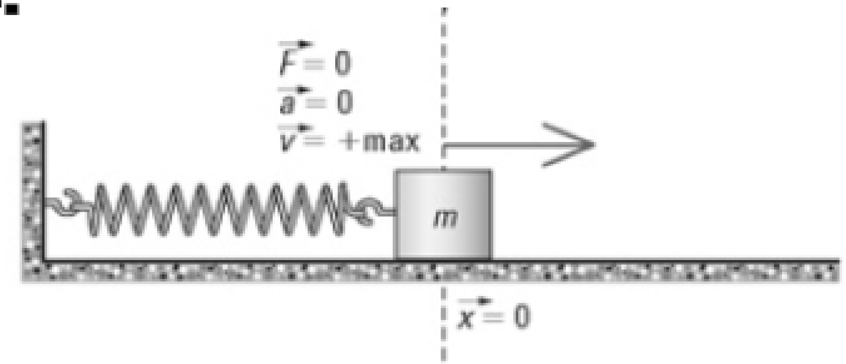
3.




The spring is fully compressed, the mass changes direction.

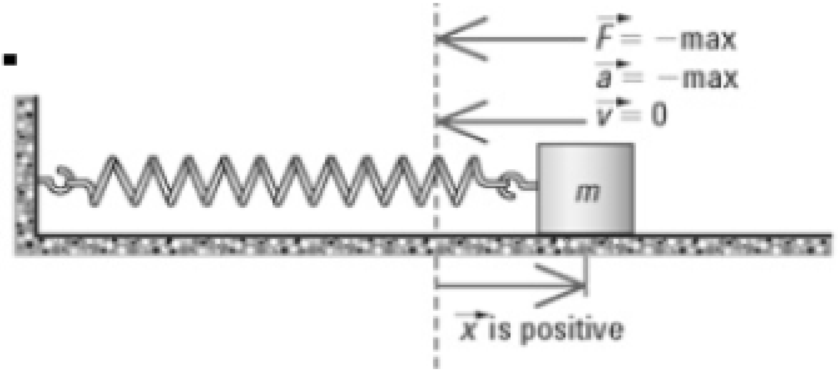
- the amount the spring is compressed is the same as the amount it was stretched by (the amplitude), but a negative value
- the restoring force in the spring is at maximum again, and the block will begin to accelerate right

**4.**

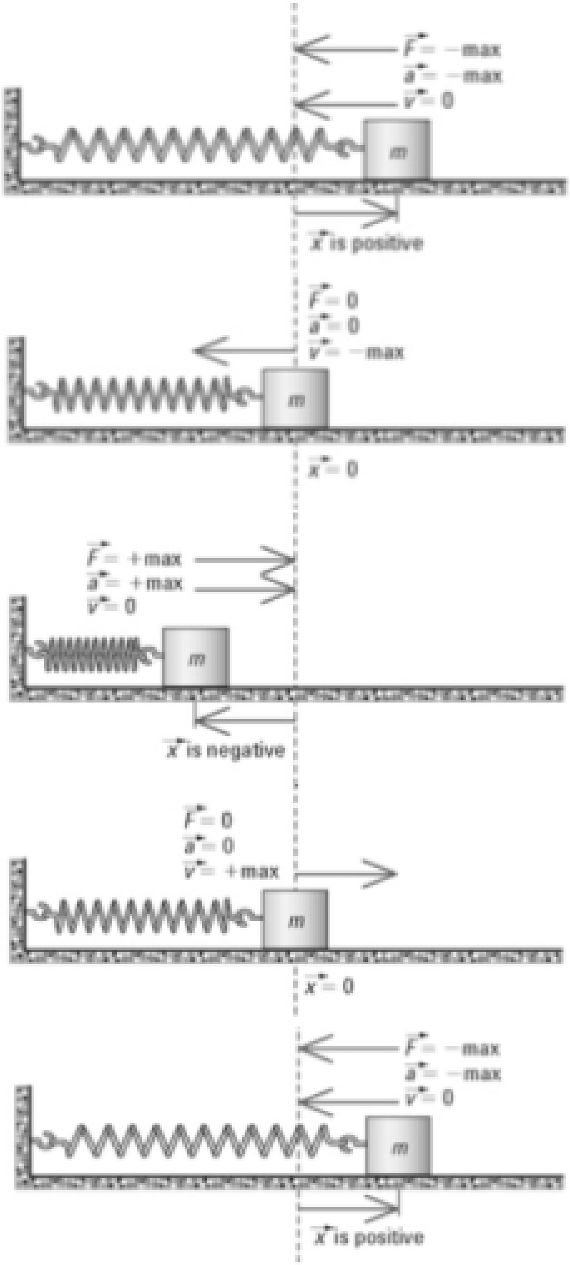



**The block continues to move right, at max. velocity.**

**5.**




**The block stretches the spring to max. amplitude and turns around: the cycle repeats.**

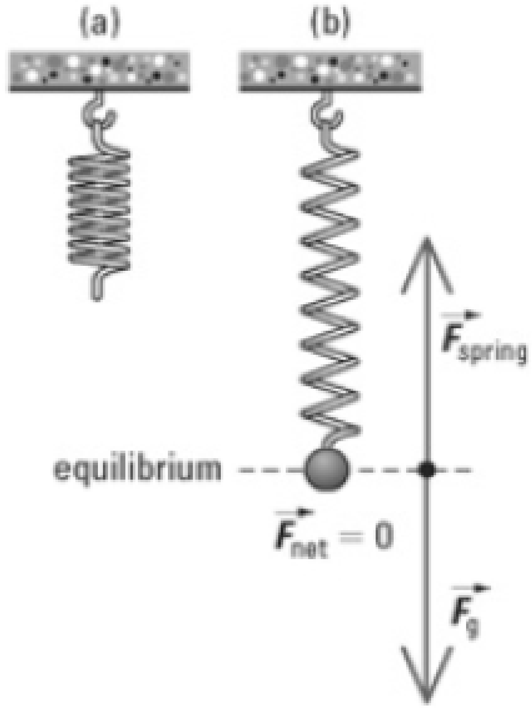



**This system has now made one complete oscillation.**

**At any point during the oscillation, the mass-spring system obeys Hooke's Law (F = kx)**

**Notice the shape the movement of the mass makes.**

**Now let's examine the same mass-spring system aligned vertically:**



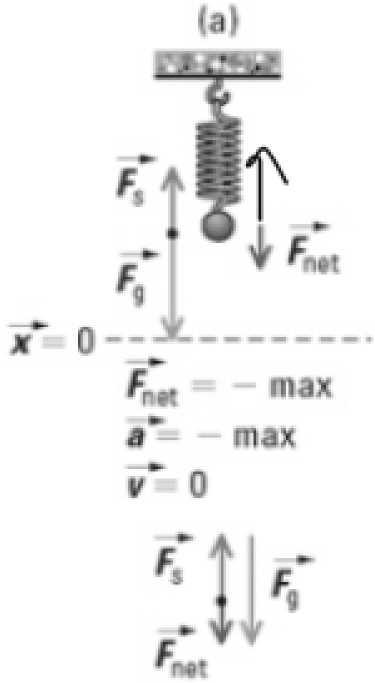

**In (a), we have a spring with no mass attached.**

**In (b), the mass is attached and stretches the spring. If the mass comes to rest, the force of gravity will be balanced by the restorative force of the spring.**

**At this equilibrium point:**

$$\vec{F}_g = \vec{F}_{spring}$$

**Let's say we push the mass up, compressing the spring a bit.**



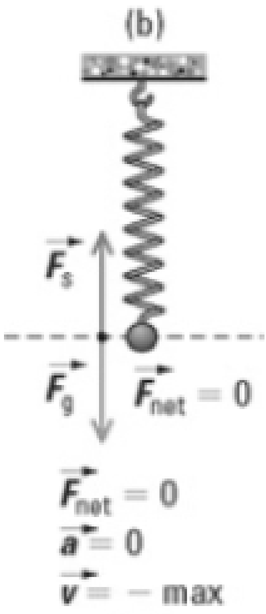

**- the mass starts at rest**

**- the force of gravity pulls down on the mass**

**- the force of the spring pulls up a small amount on the mass**

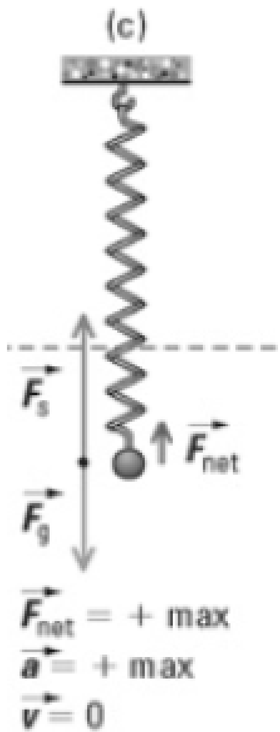
**- the overall net force is acting downward, so the mass will start to accelerate downwards**

# Later, the mass will reach equilibrium:



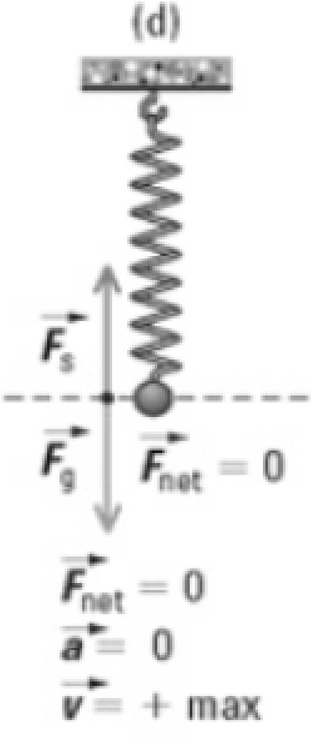

- velocity is at its maximum
- force of gravity = force of spring
- note: force of gravity does not change during the experiment!

# The mass will then travel past equilibrium...

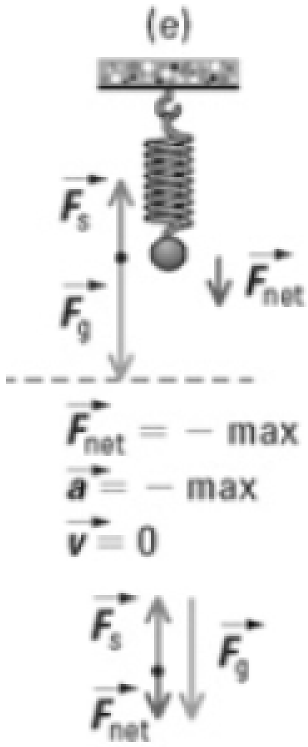



- mass comes to a stop: velocity is zero
- maximum restoring force from spring
- mass will begin to accelerate upwards
- notice: force of gravity is the same throughout the experiment!

**...before returning back to equilibrium and repeating the process.**

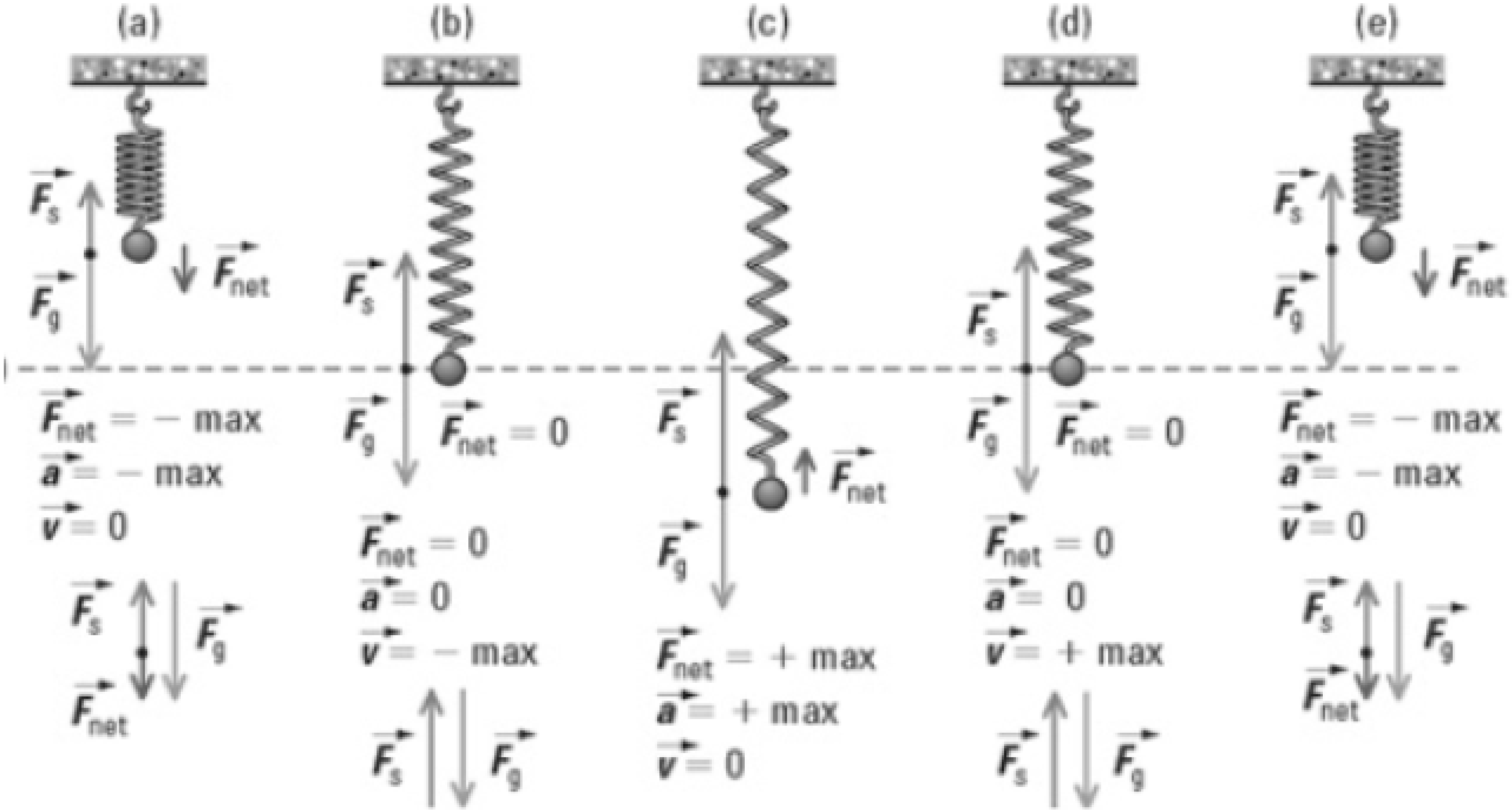



**- mass at equilibrium**



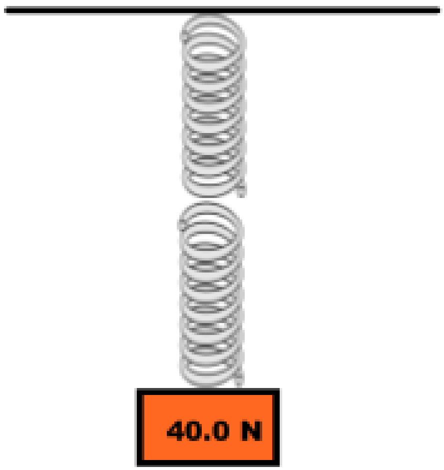

**- mass back to starting position**

# Note the pattern the mass makes as a function of time:



Understanding the conditions present in each of these situations can be helpful when solving problems involving springs and SHM.

ex) Two springs are hooked together and one end is attached to a ceiling. Spring A has a k = 25 N/m and spring B has a k = 60 N/m. A mass weighing 40.0 N is attached to the free end of the spring system. What is the total displacement of the mass?




Ans: -2.3 m

**All objects undergoing SHM follow these rules:**

- there is a restoring force acting in the opposite direction of the displacement (a force acting opposite of the movement to pull the object back and keep it oscillating)

- at the maximum displacements, the restoring force is at its maximum. This displacement is called the oscillator's amplitude. The velocity at this point is zero.

- at equilibrium, the restoring force is zero and the velocity of the object is maximum.

## Other examples of SHM:

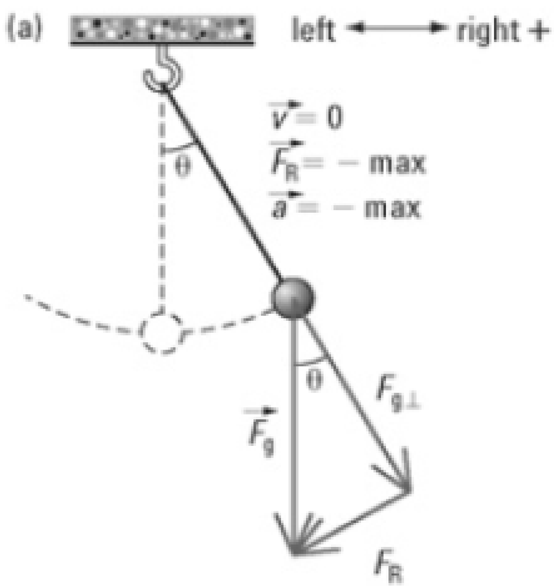
- musical instruments
- bungee jumping
- pendulums

An ideal pendulum is a good example of SHM.

**Ideal pendulums:**

- swing through a small angle
- have no friction
- have their entire mass concentrated at the bob

**The restoring force in a pendulum is a component of the force of gravity acting opposite the displacement of the bob. All simple harmonic oscillators must have a restoring force.**

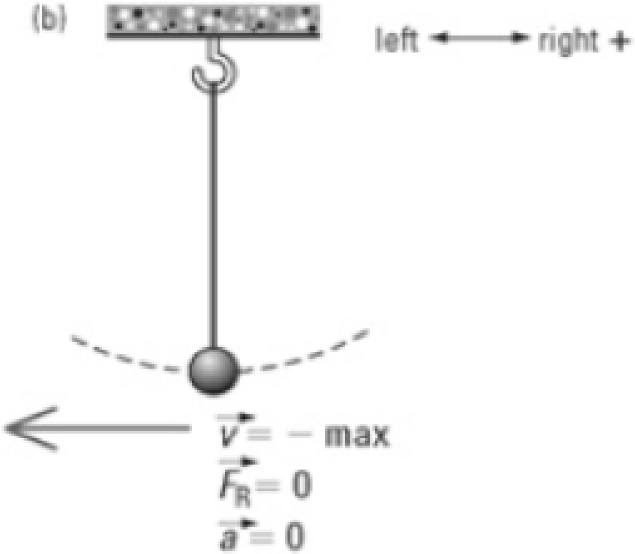



**The bob is pulled from equilibrium and released.**

- **the velocity before it begins to move is zero**
- **the restoring force is at its maximum**
- **the restoring force can be found using the equation:**

$$\vec{F}_R = \vec{F}_g \sin\theta$$

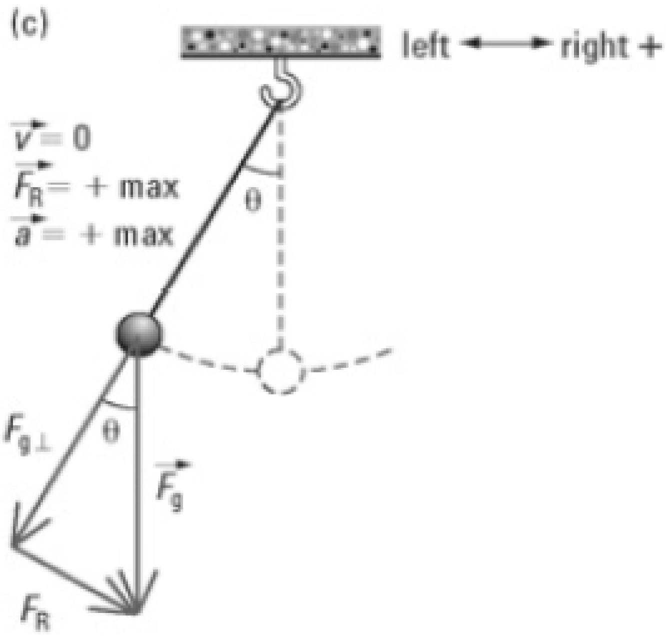
- where θ is the angle the bob is pulled back at




**Later, the bob will reach equilibrium.**

- the velocity of the bob is at its maximum
- the restoring force is zero (note the equation verifies this:

$$\vec{F}_R = \vec{F}_g \sin(0) = 0$$




**The pendulum then reaches it's other maximum displacement.**

**- the angle θ is the same as the angle the bob was released at**
**- the restoring force is the same as the original force, only in the opposite direction**

**This process continues indefinitely (so long as energy is conserved).**