Physics 20 Unit 4 - Oscillatory Motion and Mechanical Waves

Position, Velocity, Acceleration and Time of SHM

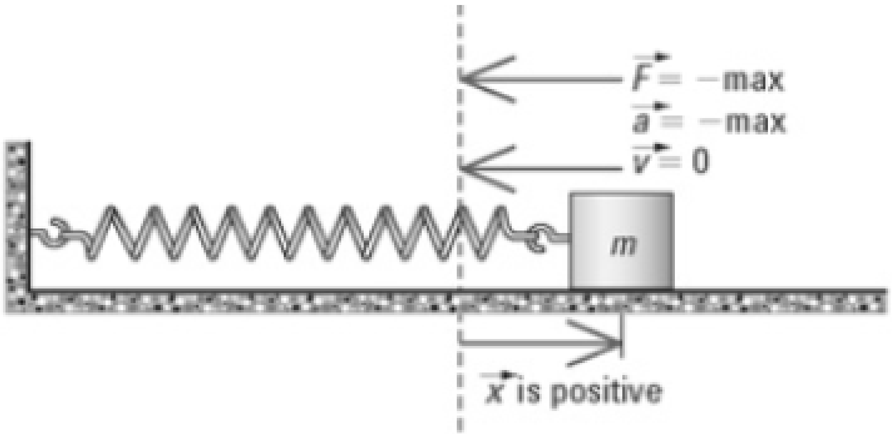
Last day, we looked at two simple harmonic oscillators:

1. a mass-spring system (horizontal and vertical)
2. a pendulum

- both were SHOs because there was a restoring force needed to keep the object in SHM
- the force and acceleration were always in opposite directions to the displacement
- we were able to determine the moments of max and min displacement, velocity, acceleration and force

Today, we will determine ways to mathematically approximate the position, velocity, acceleration and period of these types of movements.

We will consider the mass-spring system first.

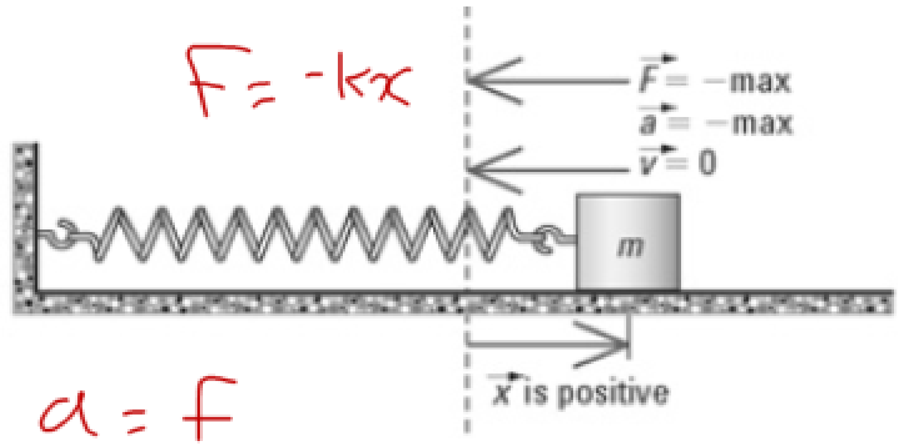


Note: these same principles would apply to a vertical mass-spring system.

1. Mass-Spring System

Finding Acceleration

$F = -kx$



We know that the maximum acceleration occurs when the mass is at its amplitude (maximum displacement).

$a = \frac{F}{m}$

But what will this acceleration be?

$a = \frac{-kx}{m}$

When the mass is at its amplitude, we can make the restoring force in the spring equal to Newton's Second Law.

$$\vec{F}_s = \vec{F}$$

$$-k\vec{x} = m\vec{a}$$

$$\vec{a} = \frac{-k\vec{x}}{m}$$

where:

$\vec{a}$ = maximum acceleration of the block (m/s^2)
$\vec{x}$ = maximum displacement of the block (m)
k = spring constant (N/m)
m = mass of block (kg)

$$\vec{a} = \frac{-k\vec{x}}{m}$$

Important Note: this equation only applies to the maximum acceleration of the mass.

In P20, we are only able to calculate the maximum acceleration on the mass. The acceleration at other points of the motion is not uniform, and is outside the scope of our course.

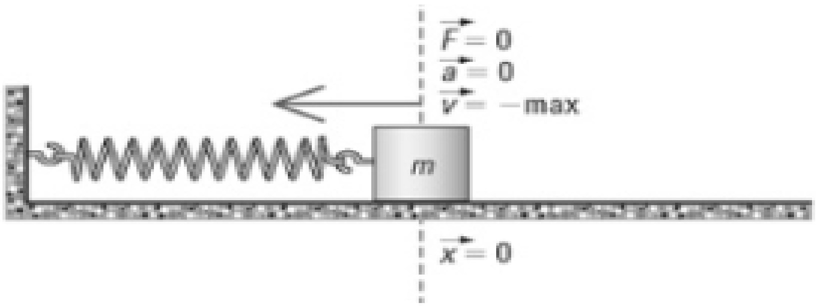
ex) In a mass-spring system, a 1.55 kg mass oscillates horizontally when attached to a spring of k = 15 N/m. If the amplitude of the oscillations is 0.75 m, what is the

a) magnitude of the maximum acceleration of the mass?

b) direction of acceleration?

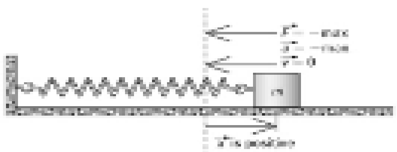
c) maximum restoring force acting on the mass?

Finding Velocity



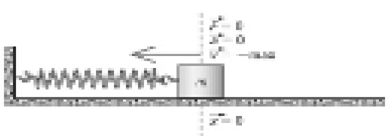
- max. velocity occurs when the mass is at equilibrium and the force is zero

We can derive this equation using the Law of Conservation of Energy.



When the mass is pulled back to its amplitude, the energy in the system is all PE_{spring}.

$$PE_{spring} = 1/2\, kx^2$$



When the mass is at equilibrium, all the PE has been turned into KE.

$$KE = 1/2 m\vec{v}^2$$

If we make these equations equal to each other:

$$E_p = E_K$$

$$1/2\ kx^2 = 1/2 m\vec{v}^2$$

$$1/2\ kx^2 = 1/2 m\vec{v}^2$$

$$\vec{v} = \vec{x}\sqrt{\frac{k}{m}}$$

***Note: the text replaces the x with an A for amplitude.**

$\vec{v}$ = maximum velocity of mass (m/s)

$\vec{x}$ = maximum displacement (amplitude) (m)

k = spring constant (N/m)

m = mass (kg)

d) In the previous mass-spring system, what will the maximum velocity of the mass be?

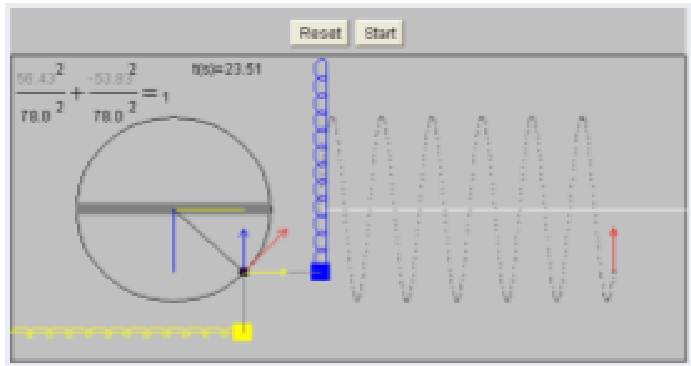
$m = 1.55\ kg$

$k = 15\ N/m$

$\vec{x} = 0.75\ m$

Finding Period

We have already seen that an object undergoing UCM can replicate an object in SHM.



http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=148

We can use this condition to determine the period of a mass-spring system, assuming

- the radius of the circle is the same as the amplitude of the
SHM
- the mass in UCM is moving with constant speed
- the periods of the UCM and SHM are the same

Recall from Unit 3 that:

$$\vec{v} = \frac{2\pi r}{T}$$

and that:

$$\vec{v} = \vec{x}\sqrt{\frac{k}{m}}$$

Therefore:

$$v = v$$

$$\frac{2\pi r}{T} = x\sqrt{\frac{k}{m}}$$

If we let x = r

$$\frac{2\pi \cancel{x}}{T} = \cancel{x}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Note: This formula DOES appear on your formula sheet!

Formula for Period of a Mass-Spring System

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where:
T = period (s)
m = mass of oscillator (kg)
k = spring constant (N/m)

Note: the period of a mass-spring system does not depend on displacement (how far it is pulled back)!

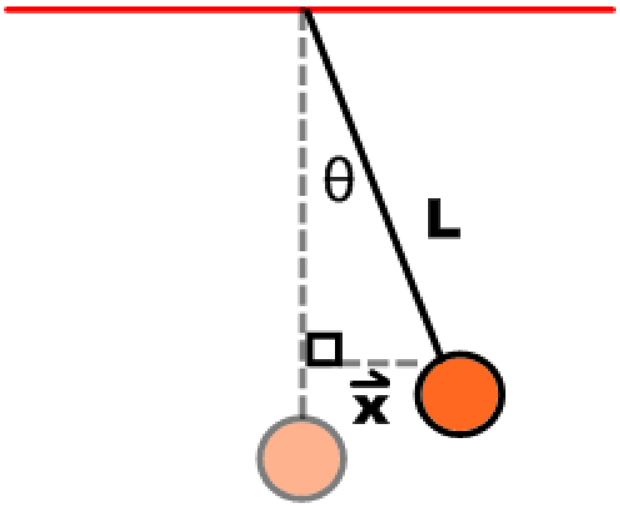
2. An Ideal Pendulum

We are only concerned with the period of a pendulum. The equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

will not work because a pendulum does not have a spring constant. To derive a formula for pendulums, we will need to do away with the k value.

To eliminate the spring constant, consider pulling a pendulum back through a small angle θ.



where:

L = length of pendulum

$\vec{x}$ = displacement

We could write that:

$$\sin\theta = \frac{\vec{x}}{L}$$

Recall that the formula for the restoring force in a pendulum is:

$$\vec{F}_R = \vec{F}_g \sin\theta$$

and that the restoring force is provided by the spring:

$$\vec{F}_{spring} = k\vec{x}$$

we can solve for $\sin\theta$ and sub-in Hook's Law:

$$\frac{\vec{F}_R}{\vec{F}_g} = \sin\theta \qquad \sin\theta = \frac{\vec{x}}{L}$$

$$\frac{\vec{F}_R}{\vec{F}_g} = \frac{\vec{x}}{L}$$

$$\frac{k\cancel{x}}{F_g} = \frac{\cancel{x}}{L}$$

$$k = \frac{m\vec{g}}{L}$$

We can now use this in our period equation.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$k = \frac{m\vec{g}}{L}$$

$$T = 2\pi\sqrt{\frac{\cancel{m}}{\frac{\cancel{m}g}{L}}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Formula for Period of a Pendulum

where:

T = period (s)
L = length of pendulum from end of string to centre of mass (m)
g = acceleration due to gravity (m/s²)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Note that the period of a pendulum does not depend on mass or amplitude.

http://learnalberta.ca/content/sep20/html/java/shm_pendulum/applet.html

The only factors effecting the period is the length and the gravitational field strength.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Pendulums are unique in that they are a very simple device which can serve a very complex purpose!

It is very easy to determine the length and period of a pendulum experimentally. If these quantities are known, one can calculate the gravitational field strength on any planet without any other equipment.

ex) An astronaut lands on the planet Langstar. To determine the acceleration due to gravity, she constructs a simple pendulum with length 5.5 m. She measures the period of the pendulum to be 6.7 s. What is the gravitational field strength on Langstar?

Ans: 4.8 m/s^2

Applications of SHM: Resonance

Objects like pendulums one have one variable effecting their SHM: length (as g is usually constant). This means that these objects have a natural oscillating frequency. This is called the objects <u>resonance frequency.</u>

Resonance Frequency - the natural frequency of vibration of an object.

Ignoring outside forces, once a SHO is set into motion, it will continue to vibrate at its resonance frequency forever.

However, in real life, friction and air resistance can change the motion of an oscillator.

In order to maintain the resonance, a small force needs to be applied: this is called the forced frequency.

Forced Frequency: when a force is added to an oscillator to keep it resonating.

An example of forced frequency is pushing a swinger on a swing set;

A small force is needed to keep the swinger in SHM. This is forced frequency. If a larger force is applied, the amplitude of the swinger increases.

Analog clocks also need a small force to keep their gears in time: this is provided by an electrically charged oscillating quartz crystal.

Resonance can also have disastrous effects:

In July 1940, the Tacoma Narrow's Bridge finished construction. It had a length of 1524 m.

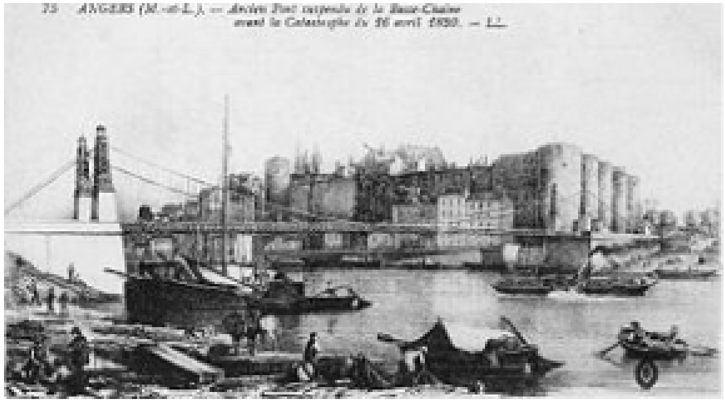
However, engineers did not account for the effect of resonance...

On Nov. 7 1940, the wind provided a small force on the bridge, causing it to vibrate. The wind was such that a force frequency was produced.

The force continued to increase slightly, increasing the amplitude of the bridge.

http://www.youtube.com/watch?v=P0Fi1VcbpAI

A similar disaster took place in 1850 in Angers, France when 478 French soldiers marched across the bridge in step, causing a forced frequency.



Resonance can also occur in large sky-scrappers, although most now have vibrating masses near their tops to counter-act these effects.