Physics 20 Unit 3 - UCM

Satellite Motion

What is a satellite?

What are some examples of satellites?

A satellite is any object which orbits around a central object (usually a planet).

We will consider two types of satellites:

1. Natural Satellites:

- **moons of a planet**
- **planets moving around the Sun**

2. Artificial Satellites:

- **communications satellites**
- **the Space Shuttle, Hubble/ISS/MIR**
- **Space Junk**

It was our old friend Newton who first devised the idea of a satellite. Recall Newton's Cannon:

He knew that if a projectile was fired with a large enough velocity, it would fall to the ground at the same rate that the Earth's surface curves. This would cause the object to be in continuous free-fall: or an orbit.

Newton knew that there would be a centripetal force acting on the cannon ball during the flight because of the ball's UCM.

Newton also knew that the manipulated variable in this experiment was the velocity of the ball.

$$\vec{F}_c = \frac{m\vec{v}^2}{R}$$

therefore:

$$\vec{v} = \sqrt{\frac{\vec{F}_c R}{m}}$$

Newton also realized that there was a force of gravity acting between the Earth and the ball:

$$\vec{F}_g = m\vec{g}$$

where m = mass of test object (the ball)

This force of gravity is what provides the centripetal force which keeps the ball in its circular orbit.

Newton determined that if he could increase the velocity of the ball just enough, the centripetal force would equal the gravitational force and the object would maintain a steady orbit (i.e. all of the centripetal force would be supplied by gravity).

Algebraically speaking:

$$\vec{F}_g = \vec{F}_c$$

$$\cancel{m}\vec{g} = \frac{\cancel{m}\vec{v}^2}{R}$$

$$\vec{v} = \sqrt{\vec{g}R}$$

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This is the speed needed for an object to enter into a steady orbit.

where: $\vec{v}$ **= escape velocity**

$\vec{g}$ **= gravitational field strength**

r =distance from the centre of the planet to the centre of the object.

***Note: this is a helpful formula which does not appear on your formula sheet! You would do well to either memorize it or know how to work it out.**

Note: that the mass of the object which we are putting into orbit does not factor into the equation to determine escape velocity.

ex) What is the escape velocity needed to put an object into orbit here on Earth?

ex) What is the escape velocity on the moon? (R = 1.74 x 10^6 m, gravitational field strength = 1.63 m/s^2)?

By a similar derivation, we can determine the speed of a satellite in orbit if we know how far it is above the surface of the planet.

$$\vec{F}_g = \vec{F}_c$$

$$\frac{G m_1 \vec{m}_2}{r^2} = \frac{m \vec{v}^2}{R}$$

*where the masses which cancel are the masses of the object.

$$\vec{v} = \sqrt{\frac{G m_e}{r}}$$

where: m_e = mass of earth (or some other planet)
r = distance from centre of planet to centre of object

***Again, note that you don't need to know the mass of the satellite to know how quickly it is moving.**

Alternatively, we can create an equation solved for T in terms of radius and mass alone.

$$\vec{v} = \vec{v}$$

$$\sqrt{\frac{\vec{G}m_e}{r}} = \frac{2\pi r}{T}$$

$$\frac{4\pi^2 r^2}{T^2} = \frac{Gm_e}{r}$$

$$4\pi^2 r^3 = T^2 G m_e$$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_e}}$$

ex) Galileo discovered 4 moons of Jupiter, listed below. Also listed is their periods of revolution and their orbital radii (centre to centre). From this data, determine the mass of Jupiter.

moon	period (days)	distance (10^6 m)
Io	1.769137786	422
Europa	3.551181041	671
Ganymede	7.154552960	1070
Callisto	16.68901840	1883

ex) When a satellite is in orbit around a planet, only one variable will effect it's speed. What is this variable? Show equations to support your solution.

ex) Mars is about half as massive as Earth. If Mars has a moon, M, with the same orbital radius as the Earth's moon, E, which satellite has a smaller period?

Set $F_c = F_g$, use period eqn and radius equation, use proportionalities.

Hint

Kepler’s Laws

Q: Who was Kepler?

- **Johannes Kepler was a mathematician and astronomer who worked for the great Tycho Brahe.**
- **Who was Tycho Brahe?**

Tycho Brahe

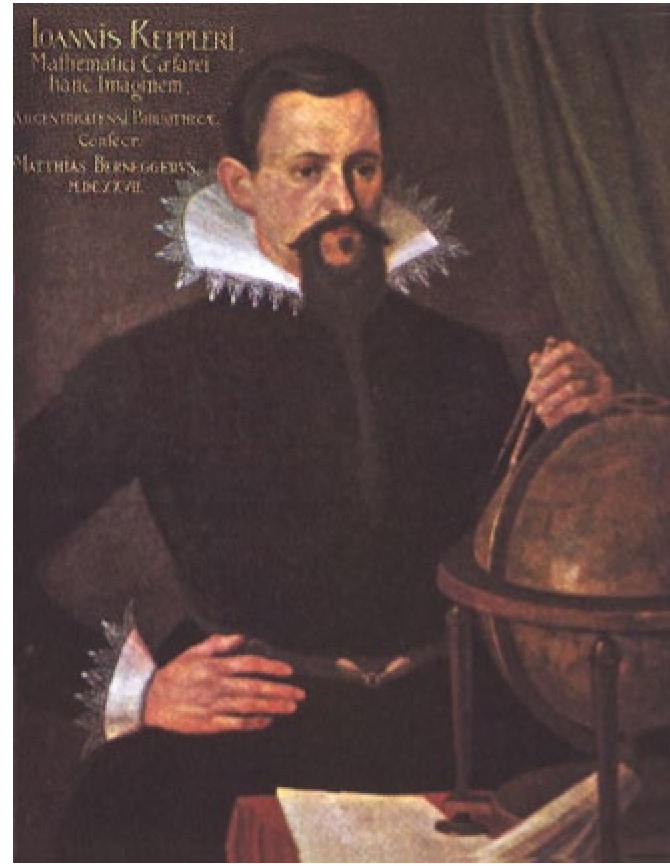
- **Brahe was a Danish astronomer remembered for three things:**

  1) Made incredibly detailed observations of the planets. Collected more data and better data than anyone ever before him.

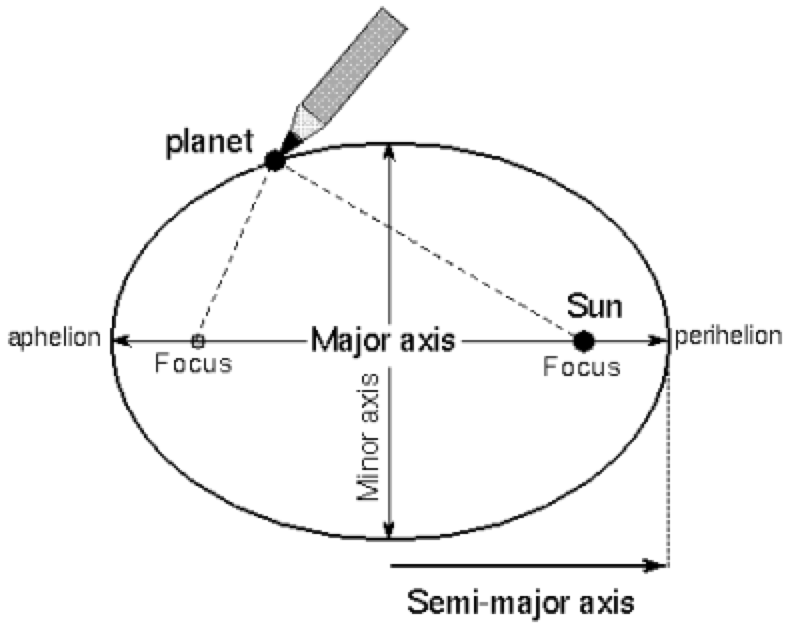
  2) Was given a sweet crib by the King of Denmark on his own island so he could make more sweet observations.

  3) Liked to drink, get into arguments and fight with broadswords, resulting in his nose being cut off. A replacement nose was fashioned out of bronze, which he continuously rubbed with oil. He had a gold nose made for special occasions.

- **Kepler was lucky enough to work with Brahe and figure out his lifetime of measurements and observations.**
- **From these observations, Kepler came up with three laws:**



Kepler's First Law:

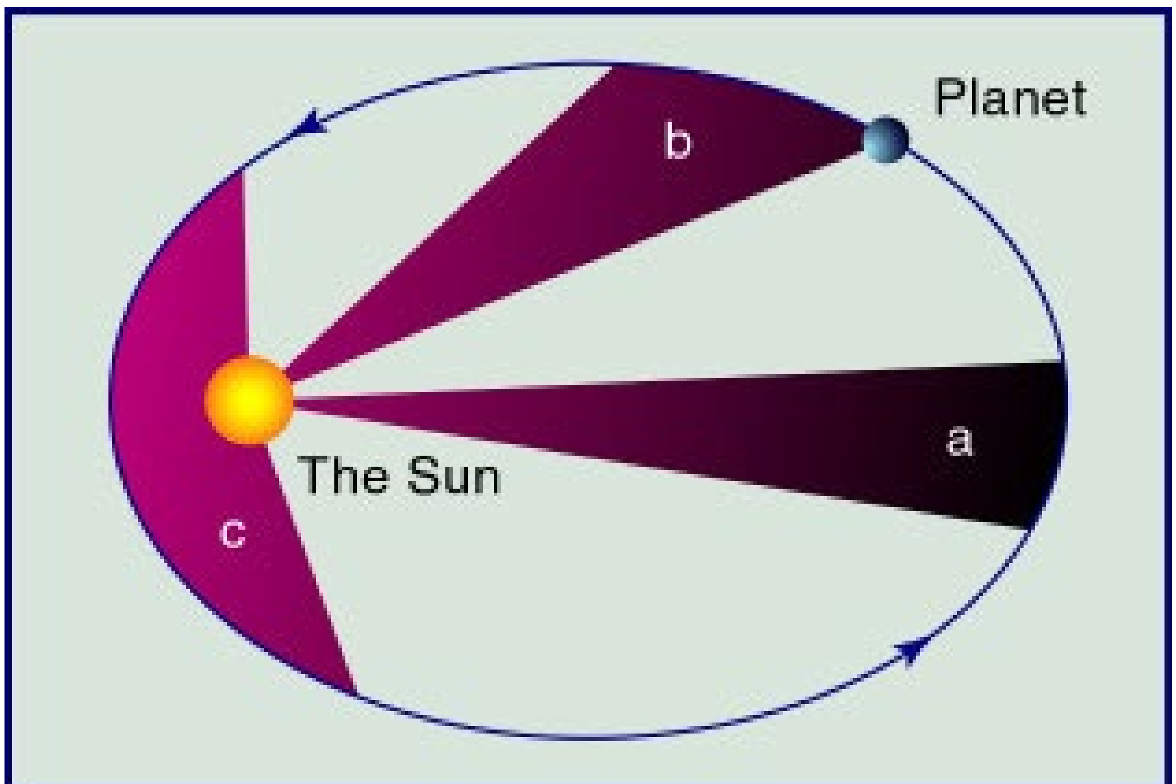


Drawing an **ellipse**: loop string around thumb tacks at each **focus** and stretch string tight with a pencil while moving the pencil around the tacks. The Sun is at one focus.

- **Planets move in ellipses, with the sun at one focus**
- **Unlike all earlier astronomers, Kepler found that planets moved around the sun in an ellipse, not a circle. The radius of the orbit in an ellipse is equal to the semi-major axis.**

Kepler's Second Law:

- Planets sweep out equal areas in equal times (i.e. their orbital speed is not fixed)



Kepler's Third Law:

- The square of the period of a planet's orbit divided by the cube of its orbital radius is a constant.
- Kepler was able to determine this equation based on Brahe's excellent measurements.

$$\frac{T^2}{r^3} = K$$

where:
T = period
r = radius
K = a constant

In the equation, the constant K changes for each planet.

- **Kepler's third law equation was proven experimentally. It matched perfectly the data that Brahe collected. It's about as 'true' as we can get in Physics.**
- **But, while it worked, Kepler had no idea why it worked. He thought the answer was magnetism.**
- **So how does all this tie into UCM?**

Newton and Kepler

- Many years after Kepler, Newton determined that it was gravity that kept the planets in their orbits. His idea that the centripetal force was supplied by gravity proved Kepler's Third Law!
- This is experimental proof that Newton's ideas for centripetal motion work out!

$$\vec{F}_c = \vec{F}_g$$

$$\frac{4\pi^2 r \cancel{m}}{T^2} = \frac{G \cancel{m_1} m_2}{r^2}$$

$$\frac{4\pi^2}{Gm} = \frac{T^2}{r^3}$$

a constant

Kepler's 3rd Law