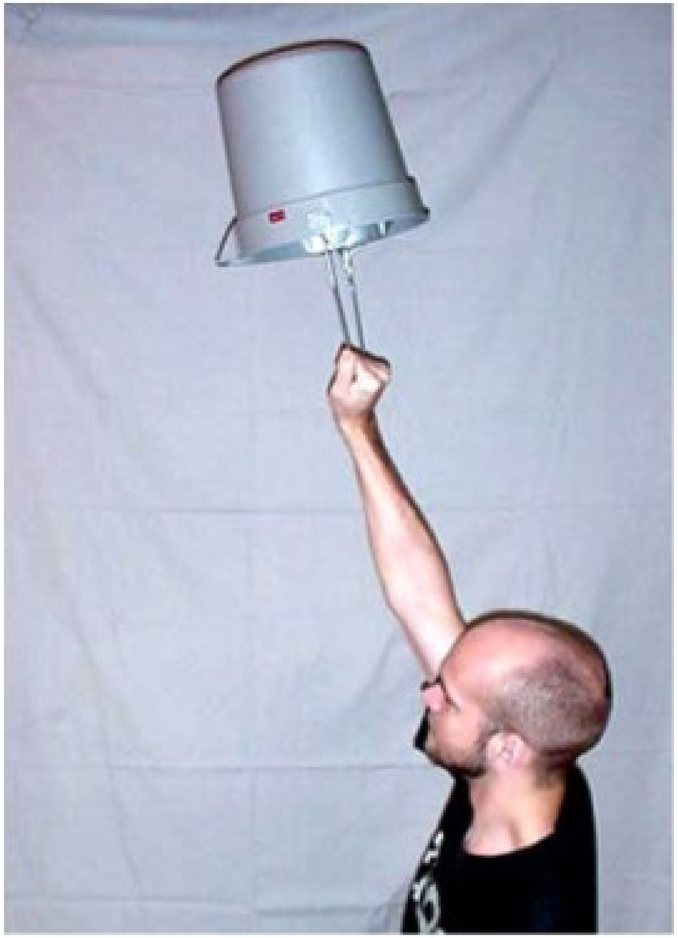
# Everything you wanted to know about UCM and more!



This unit deals with three ideas we've already spent some time considering:

- kinematics
- dynamics
- gravity

However, up until now, we've only considered objects moving in straight lines. The physics of circles still remains a mystery to you.

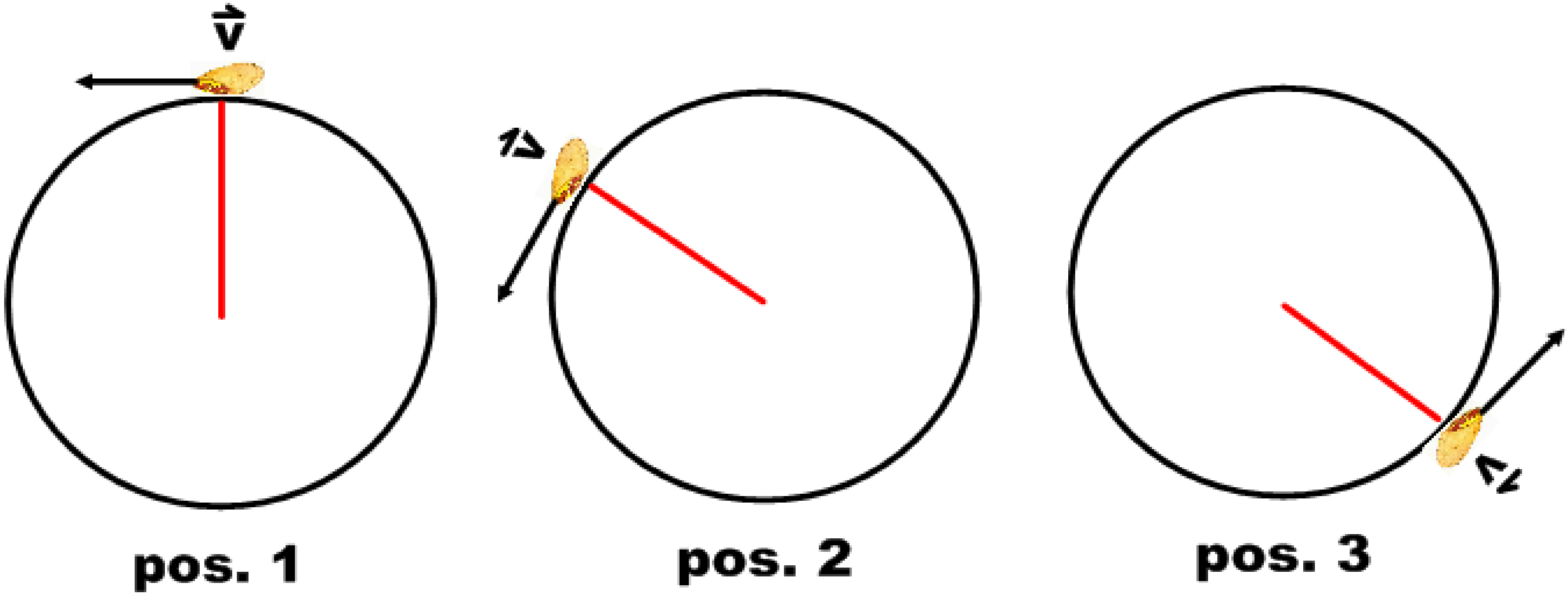
# Uniform Circular Motion

***Uniform*** **- Meaning the same, constant. A basketball team wears "uniforms" to all look alike.**
***Circular*** **- Like, circles and stuff. Things that move in a circle.**
***Motion*** **- to move…you get the picture.**

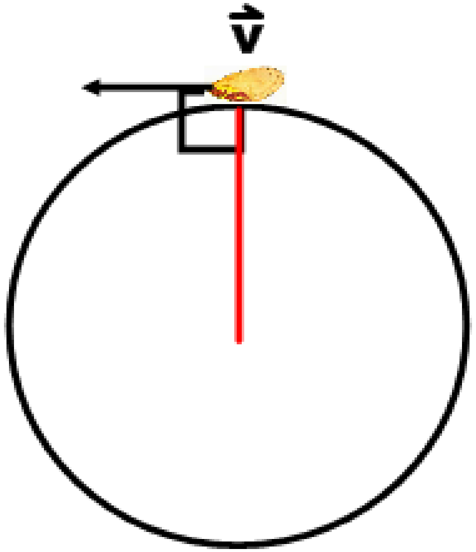
**Uniform circular motion can be found all around us. Tying a rope to, say, a hot pocket, and swinging that hot pocket around in a circle at a constant speed constitutes UCM. As long as the speed stays the same, the object is in UCM.**

**Notice I said speed, not velocity...**

**Lets draw a picture of this hot pocket on a string.**




**At all three points in the hot pocket's path, the magnitude of the velocity is the same (as the motion is uniform...therefore the pocket maintains a constant speed). However, you can see that the direction of the velocity is constantly changing depending on where the object is in its circular path.**




**The velocity of the object in UCM always acts at a tangent to the circle in the direction of movement. The velocity at any given point in time (called the instantaneous velocity) is also perpendicular to the radius of the circle.**

**Direction of velocity is perpendicular to the radius, at a tangent to the circle!!**

**So, speed stays the same, but velocity is always changing. Hmm... a changing velocity? What does that mean?**

**Recall that in Physics, "a change in" can be represented by the Greek symbol delta (Δ). So we could say that in UCM, we have a $\Delta\vec{v}$. What formulas do we have that involve $\Delta\vec{v}$?**

# Good Old

$$\Delta\vec{v} = \Delta\vec{d} \,/\, \Delta t$$

**Now, the value Δt stands for the change in time, in this case, the amount of time it takes for the object to make one complete revolution around in a circle.**

**In UCM, this time value is called the period, which is represented by a capital letter T.**

**A period is the amount of time it takes an object to complete one cycle of movement.**

**So, if we wanted to determine the magnitude of the velocity of our hot pocket, we could substitute in T for Δt.**

$$\Delta\vec{v} = \Delta\vec{d} \,/\, T$$

**But what is Δd? This variable represents the distance the object goes through in its circular path. This displacement is the same as the circumference of the circle which the object traces out.**

$$\vec{\Delta} d = 2\pi r$$

**We can now write an equation for the velocity of an object moving in a circle of radius r, in a period T…**

$$\Delta\vec{v} = 2\pi r / T$$

**where** $\vec{v}$ **= velocity (m/s)**

**r = radius of circle (m)**

**T = period of motion (s)**

This formula can also be found on your formula sheet under the "Kinematics" section.

**Example: If the Hot Pocket has a period of 1.5 s and the string is 1.25 m long, what is the magnitude of the HP's velocity?**

**Ex: A jet flys at a height of 10 000 m above the surface of the earth in a circular path around the planet. If the velocity of the jet is 150 m/s, how long does it take the jet to go around the world?**

# Centripetal Acceleration

**Now, we said that the direction of the velocity is always changing. A changing velocity must mean we have some sort of acceleration. How can we derive a formula for acceleration?**

**(Warning! The following derivation is kinda tricky and uses some interesting math. If you're having trouble with this, don't panic. You just need to know the general jist of the thing and the final equation.)**

The acceleration of an object in UCM can be found by

$$\vec{a}_c = \vec{v}^2 / R$$

Where: $\vec{a}_c$ = centripetal acceleration ($m/s^2$)

$\vec{v}$ = velocity (m/s)

R = radius (m)

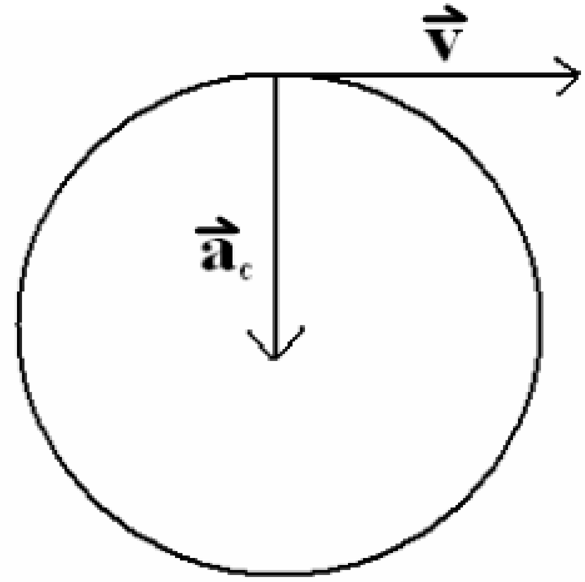
(I use a capital here because the formula sheet switches to a capital as well)

Hey, where did the word "centripetal" come from? It was actually coined by our pal Newton, when he worked out the physics of circular motion hundreds of years ago.

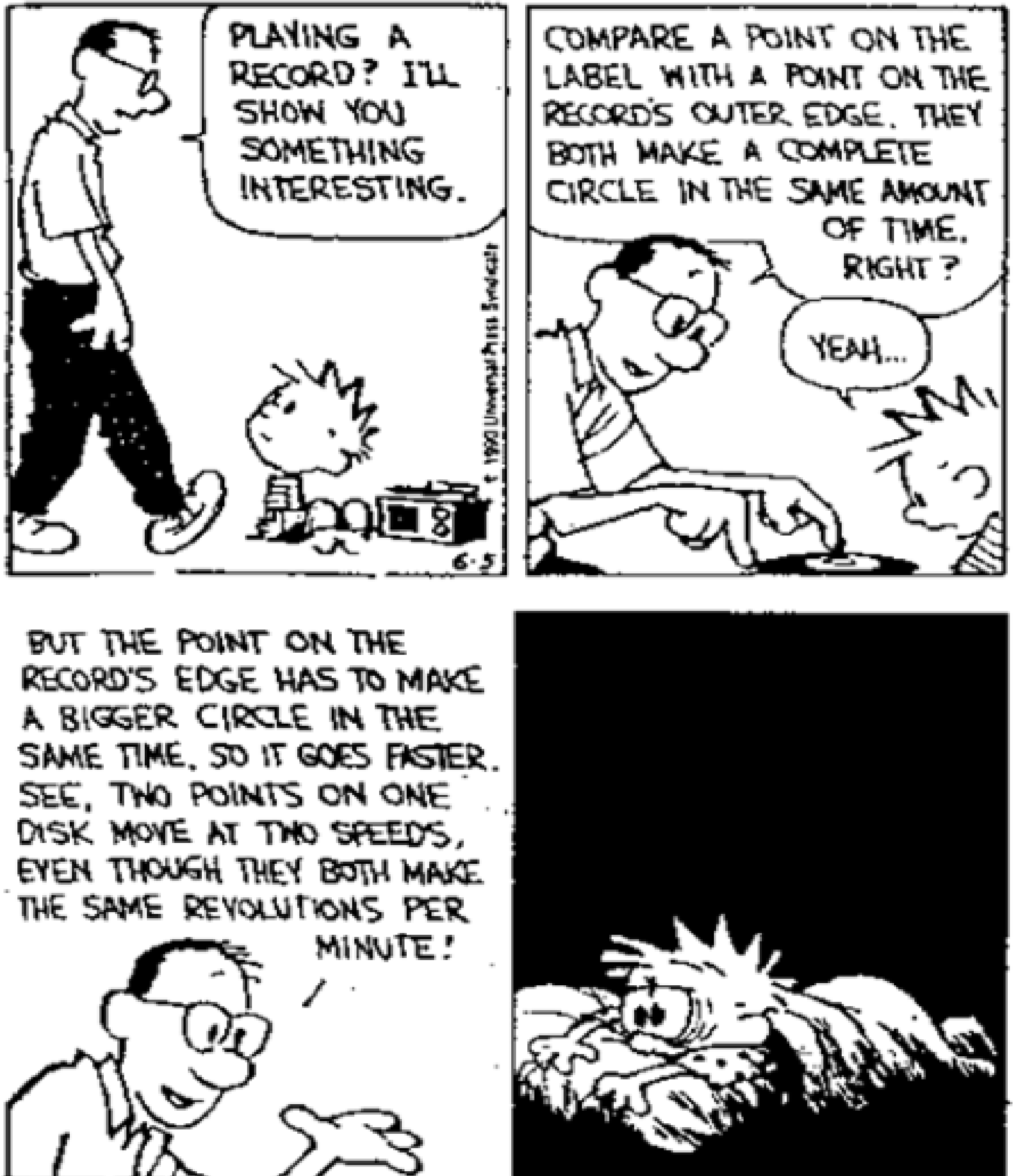
Centripetal means "centre seeking".

He called it centripetal acceleration because in UCM, the acceleration always points to the centre of the circle.

***Draw Me!**




**So velocity acts at a tangent to the circle, perpendicular to the radius, and the acceleration always acts towards the centre of the circle!**


PLAYING A RECORD? I'LL SHOW YOU SOMETHING INTERESTING.
COMPARE A POINT ON THE LABEL WITH A POINT ON THE RECORD'S OUTER EDGE. THEY BOTH MAKE A COMPLETE CIRCLE IN THE SAME AMOUNT OF TIME. RIGHT?
YEAH...
BUT THE POINT ON THE RECORD'S EDGE HAS TO MAKE A BIGGER CIRCLE IN THE SAME TIME, SO IT GOES FASTER. SEE, TWO POINTS ON ONE DISK MOVE AT TWO SPEEDS, EVEN THOUGH THEY BOTH MAKE THE SAME REVOLUTIONS PER MINUTE!

**Ex: A car takes a curve of radius 15 m at 45 km/h. What is it's acceleration?**

# Centripetal Force

**By now, if you've learned anything in my class, you've learned Newton's second law: where there's acceleration, there's a force. This force is called the centripetal force.**

**To find centripetal force, we use Newton's second law.**

Given: $$\vec{a}_c = \vec{v}^2 / R$$

and as: $$\vec{F} = m\vec{a}$$

If we multiply the centripetal acceleration equation by mass on each side, we will get a new equation for centripetal force:

$$\boxed{\vec{F}_c = \frac{m\vec{v}^2}{R}}$$

where: $R$= radius (m)

$\vec{v}$ = velocity (m/s)

$m$ = mass (kg)

$\vec{F}_c$ = centripetal force (N)

**We can also express force in terms of T,**

$$\vec{F}_c = m\vec{v}^2 / R \qquad \text{as } \Delta\vec{v} = 2\pi r / T$$

$$\vec{F}_c = m(2\pi r / T)^2 / R$$

$$\boxed{\vec{F}_c = \frac{4\pi^2 Rm}{T^2}}$$

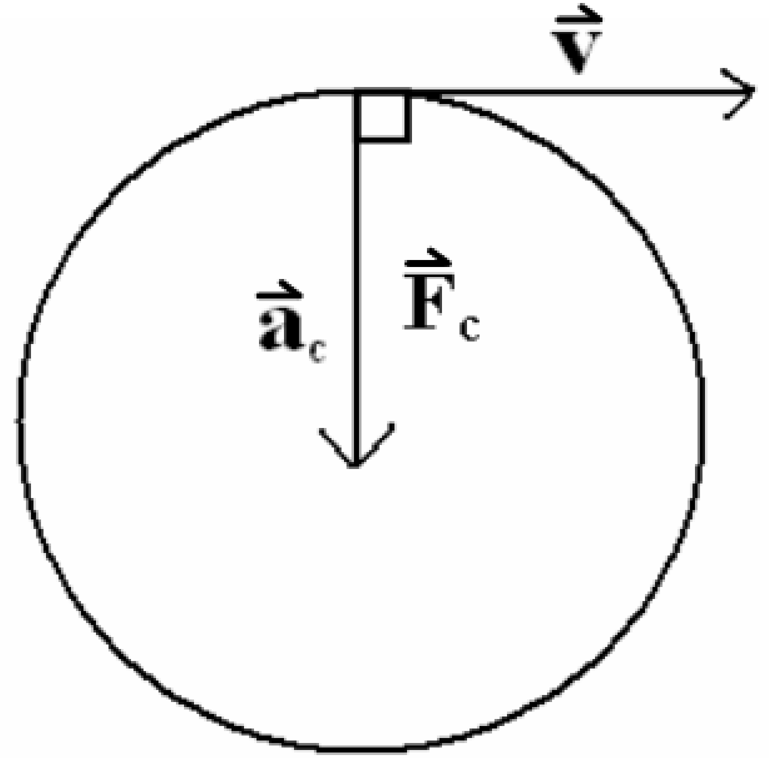
**where:** $R$ = radius (m)

$M$ = mass (kg)

$T$ = period (s)

$F_c$ = centripetal force

Note that for this equation, we don't need to know the velocity of the object.




By Newton's second law, the direction of the centripetal force is the same as the direction of the centripetal acceleration.

Therefore, in UCM, there is a force due to centripetal motion pointed towards the centre of the circle.

**This centripetal force can be supplied by any different force: centripetal force can be friction, tension, gravity...whatever.**