The Kinematics Equations

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Kinematics is the study of motion without much regard for what is causing the motion.

We can develop 3 new equations for studying objects with a constant acceleration. All we need to do is substitute old equations to form new ones.

So let's recycle us some new equations!

1. Displacement from velocity and time.

During constant acceleration, velocity is changing, so we can no longer talk about plain old $\vec{v}$ in the equation

$$\vec{d} = \vec{v}t$$

anymore.

For problems with a constant acceleration, we must use the average velocity, $\vec{v}_{ave}$.

1 $v_{ave} = 1/2\,(\vec{v}_f + \vec{v}_i)$ **where** $\vec{v}_f$ **= final velocity**

$\vec{v}_i$ **= initial velocity**

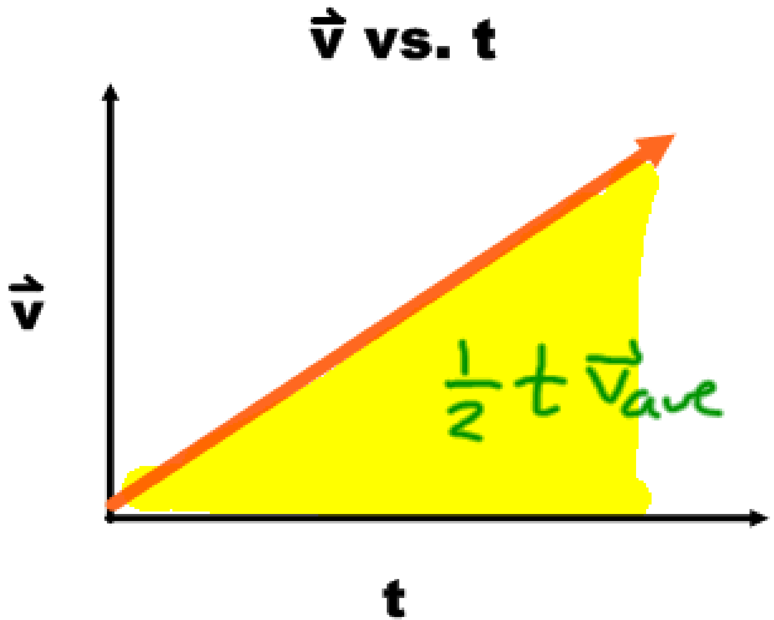
Recall

2 $\vec{d} = \vec{v}t$

Sub 1 into 2

$$\vec{d} = 1/2\,(\vec{v}_f + \vec{v}_i)t$$

This eqn. makes sense if we consider a velocity vs time graph of constant acceleration.



Recall the area under this graph gives displacement.

The area under this graph forms a triangle.

$$A_{\triangle} = 1/2bw$$

This is where the 1/2 comes from

ex) A puma moves with a velocity of 3.00 m/s E and accelerates constantly. If the velocity after 4.70 s is 15 m/s E, what is the displacement of the object?

Ans: 42 m E

ex) A driver accelerates constantly to a velocity of 7.5 m/s during 4.5 s. The driver's displacement is 19 m E.

What is the v_i?

Ans: 0.94 m/s

2. Displacement when acceleration and time are known.

Sometimes, problems will give us $\vec{a}$, t, $\vec{v}_i$ but no $\vec{v}_f$ (and sometimes there's no $\vec{v}_i$, but more on that later...). But we can still solve for $\vec{d}$ ... first we must combine two equations from the past...

If $\quad$ **1** $\quad \vec{d} = 1/2\,(\vec{v}_f + \vec{v}_i)t$

but we don't have $\vec{v}_f$, so recall:

$\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{t}$ therefore: **2** $\quad \vec{v}_f = \vec{v}_i + \vec{a}t$

sub **2** into **1**

$$\vec{d} = 1/2\,((\vec{v}_i + \vec{a}t) + \vec{v}_i)t$$

$$= 1/2\,(2\vec{v}_i + \vec{a}t)t$$

$$\vec{d} = \vec{v}_i t + 1/2\vec{a}t^2$$

$$\vec{d} = \vec{v}_i t + 1/2\vec{a}t^2$$

Remember that these equations only work for a constant acceleration.

ex) A biker passes a lightpost at the top of a hill traveling at 4.5 m/s. She accelerates down the hill at a constant rate of 0.40 m/s^2 for 12.0 s. How far down the hill did she move?

Ans: 83 m down the hill.

ex) A sheep starts from rest and accelerates at a constant rate 3.15 m/s^2 forward for 28.65 s. What is the displacement during this time?

v_i = 0 because the sheep starts from rest.

Secret Sheep Thing

Ans: 1.29 x 10^3 m forwards.

3. Displacement, velocity, and acceleration are known (the no time eqn).

If $\vec{a}$ and $\vec{v}$ are known, but t is not, we can still solve for $\vec{d}$...

1 $\vec{d} = 1/2\,(\vec{v}_f + \vec{v}_i)t$

$\vec{v}_f = \vec{v}_1 + \vec{a}t$

2 $t = \dfrac{\vec{v}_f - \vec{v}_1}{\vec{a}}$

Sub 2 into 1 ...

$$\vec{d} = 1/2\,(\vec{v}_f + \vec{v}_i)\frac{(\vec{v}_f - \vec{v}_1)}{\vec{a}} = t$$

rearranged

$$\vec{d} = \frac{1}{2\vec{a}}(\vec{v}_f + \vec{v}_i)\,(\vec{v}_f - \vec{v}_1)$$

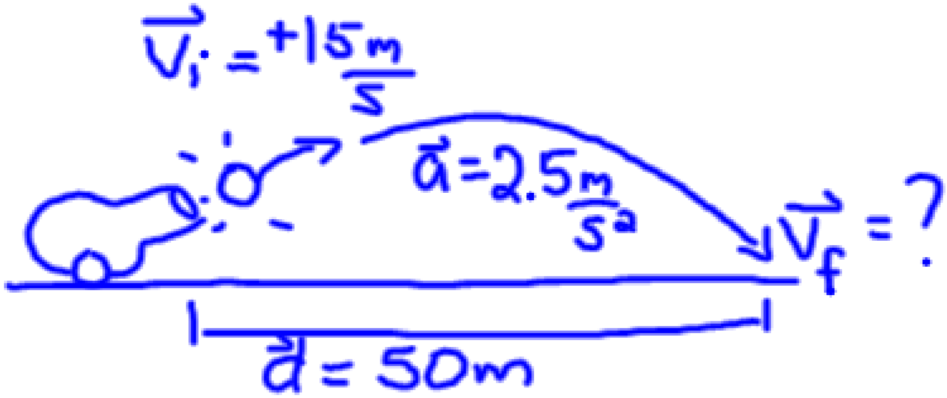
$$\vec{d} = \frac{1}{2\vec{a}}(\vec{v}_f^{\,2} + \cancel{\vec{v}_1\vec{v}_f} - \cancel{\vec{v}_1\vec{v}_f} - \vec{v}_i^{\,2})$$

$$\vec{d} = \frac{1}{2\vec{a}}(\vec{v}_f^{\,2} - \vec{v}_i^{\,2})$$

Therefore:

$$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\vec{d}$$

ex) **A cannon is shot with initial velocity of +15 m/s. The cannon ball travels for 50 m with a constant acceleration of 2.5 m/s^2. What is the final velocity of the ball?**



Ans: +22 m/s